K25P 2019

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Supplementary) Examination, April 2025
(2021 and 2022 Admissions)
MATHEMATICS
MAT 2C 09 : Foundations of Complex Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Attempt **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

1. Does the set $D = \{z \in C : 1 < |z| < 2\}$ is simply connected ? Justify your answer.
2. Given that $\gamma(t) = -1 + 4e^{6\pi it}$, $0 \leq t \leq 1$. Find the index of γ with respect to the point 2.
3. Prove that the function $f(z) = z^4 e^{\frac{-3}{4z^2}}$ has an essential singularity at z = 0.
4. Prove that an entire function has a removable singularity at infinity if it is a constant.
5. Define the function $E_p(z)$ for p = 0, 1, . . . Show that $E_p(z/a)$ has a simple zero at z = a.
6. Prove the following : If Re $z_n > 0$, then the product $\prod z_n$ converges absolutely if and only if the series $\sum(z_n - 1)$ converges absolutely.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks. **(4×16=64)**

Unit – I

7. a) Let G be a region and let f and g be analytic functions on G such that $f(z)g(z) = 0$ for all $z \in G$. Prove that either $f \equiv 0$ or $g \equiv 0$.
   b) State and prove Morera's theorem.
   c) Prove the following :
   If $\gamma : [0, 1] \to C$ is a closed rectifiable curve and $a \notin \{\gamma\}$, then $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is an integer.

P.T.O.

8. Prove the following :

   Let G be a connected open set and let $f : G \to C$ be an analytic function. Then the following conditions are equivalent.

   a) $f \equiv 0$;

   b) there is a point a in G such that $f^n(a) = 0$ for each $n \geq 0$;

   c) $\{z \in G : f(z) = 0\}$ has a limit point in G.

9. a) State and prove Cauchy's integral formula (First Version).

   b) Find all possible values of $\int_\gamma \frac{dz}{1+z^2}$ where γ is any closed curve in C not passing through $\pm i$.

Unit – II

10. a) State and prove Schwarz lemma.

    b) Prove the following :
    If f has an isolated singularity at z = a, then the point z = a is a removable singularity if and only if $\lim_{z \to a}(z-a)\,f(z) = 0$.

11. a) State and prove the Rouche's theorem.

    b) State and prove the Argument principle.

    c) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} = \frac{\pi}{\sqrt{2}}$.

12. State and prove the Laurent Series Development.

Unit – III

13. State and prove Arzela-Ascolis theorem.

14. a) With the usual notations, prove that $C(G, \Omega)$ is a complete metric space.

    b) State and prove Montel's theorem.

15. State and prove the Riemann Mapping Theorem.